implemented easily on a pocket calculator. In terms of accuracy, the described formulas represent a considerable improvement on the foundation of coaxial component design and are fully compatible with the needs of modern computer-aided microwave coaxial circuit design.

References

[1] M. A. Gunston, *Microwave Transmission Line Impedance Data*. New York: Van Nostrand Reinhold, 1972.

[2] S. Frankel, *Multiconductor Transmission Line Analysis*. Dedham, MA: Artech House, 1978.

[3] H A. Wheeler, "Transmission-line conductors of various cross section," *IEEE Trans. Microwave Theory Tech.*, vol MTT-28, pp. 73–83, Jan. 1980.

[4] H. A. Wheeler, "Transmission-line properties of a round wire in polygon shield," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 712–721, Aug. 1979.

[5] H. J. Riblet, "An accurate determination of the characteristic impedance of the coaxial system consisting of a square concentric with a circle," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 714–715, Aug. 1975.

[6] H. J. Riblet, "An accurate approximation of the impedance of a circular cylinder concentric with an external square tube," *IEEE Microwave Theory Tech.*, vol. MTT-31, pp. 841–844, Oct 1983.

[7] W. Lin, "A critical study of the coaxial transmission line utilizing conductors of both circular and square cross section," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 1981–1988, Nov 1982

[8] W. Lin, "Polygonal coaxial line with round center conductor," *IEEE Trans. Microwave Theory Tech.*, pp. 545–555, June 1985.

[9] P A. A. Laura and L. E. Luisoni, "Approximate determination of the characteristic impedance of the coaxial system concentric with a circle," *IEEE Trans. Microwave Theory Tech*, vol. MTT-25, pp 160–161, Feb 1977.

[10] P. A. A. Laura and L. E. Luisoni, "An application of conformal mapping to the determination of the characteristic impedance of a class of coaxial systems," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 162–163, Feb. 1977.

[11] T. K Sheshadri and K. Rajaian, "Accurate estimation of characteristic impedance of coaxial transmission-line problems by the eigenfunction approach," *Proc. IEEE*, vol. 70, pp. 82–83, Jan. 1982.

[12] T. K. Sheshadri and K. Rajaian, "Eigenfunction solution of a class of TEM transmission line," *Proc. Inst. Elec. Eng.*, pt. H, vol. 131, pp. 279–280, Aug. 1984.

[13] L. N. Epele, H. Fanchiotti, C. A G. Canal, and H. Vucetich, "Characteristic impedance of coaxial lines bounded by *N*-regular polygons," *Proc. IEEE*, vol. 72, pp. 223–224, Feb. 1984.

[14] S. G. Pan, "A method of solving coaxial transmission lines of the complication cross-section," *Scientia Sinica*, series A, pp. 205–217, Feb. 1987.

[15] S. G Pan, "Approximate determination of the characteristic impedance of the coaxial system consisting of an irregular outer conductor and a circular inner conductor," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 61–63, Jan. 1987.

[16] L. V Kantorovitch and V. Krylov, *Approximate Methods of Higher Analysis*. New York. Interscience Publishers, 1958

[17] F. Oberhettinger and W. Magnus, *Applications of Elliptic Functions in Physics and Technology*. New York: Springer, 1949.

Mode Stability of Radiation-Coupled Interinjection-Locked Oscillators for Integrated Phased Arrays

KARL D. STEPHAN, MEMBER, IEEE, AND SONG-LIN YOUNG

Abstract —**An array of coupled oscillators can synthesize the microwave phase relationships needed for phased arrays by means of a technique known as interinjection locking. The mode required must be stable, and a general approach for evaluating mode stability and predicting frequency and phase relationships is applied to an experimental two-element 10 GHz array. Radiation coupling between the two oscillators leads to coherent operation, and the simple theory developed successfully predicts the system's behavior over a wide range of interoscillator distances.**

Manuscript received July 20, 1987; revised November 28, 1987. This work was supported by the U.S. Army Research Office under Contract DAAL03-86-K-0087

The authors are with the Department of Electrical and Computer Engineering, University of Massachusetts, Amherst, MA 01003.

IEEE Log Number 8819969.

I. Introduction

Recently a novel method of phase generation and control for phased arrays has been developed. The technique, described as interinjection locking [1], [2] or parasitic injection locking [3], consists in driving each element in a phased antenna array with its own directly coupled oscillator. Suitable coupling between the antenna elements causes the system as a whole to run coherently, synthesizing the properly phased drive for each element. Rather than requiring a phase shifter for each element, the system can be steered by a few externally controlled injection inputs at strategic points. The difficulties and losses encountered when a phase-shifting circuit must be provided for each element could be greatly reduced by applying this technique to microwave, and especially millimeter-wave, integrated circuits in which phase shifter losses rise rapidly with increasing frequency.

A system of N nonlinear oscillators can in principle operate in any one of N single-frequency modes, and even more if multiple-frequency operation is considered. Typically only one of these modes meets the phased array requirements, so some means must be established for evaluating mode stability in systems of coupled oscillators. In this paper we analyze two oscillators coupled solely by means of the free-space interaction between their respective antenna elements. The oscillators are modeled as energy-storing $L-C$ tank circuits in parallel with voltage-dependent negative conductances. A simplified far-field slot antenna model is used to derive the mutual admittance of the two antennas. Even-odd mode analysis yields the normal modes of the system, and a theorem from averaged potential theory is used to determine which mode is stable. Two microstrip Gunn diode oscillators were built to verify the essential features of the model. Oscillator frequencies, relative phases, and radiation patterns were measured as functions of the interantenna distance, and the periodic alternation of modes with distance predicted by theory was confirmed quite well. Although the small system studied is of limited practical use, it has many features in common with larger practical interinjection-locked systems.

II. Theory

Most studies of multiple-device oscillators, such as Kurokawa's [4], assume that the primary energy storage mechanism is a resonant-structure mode common to all the devices, with relatively little energy stored within each device's associated circuitry. In contrast to this, the interinjection-locking approach begins with self-sufficient oscillators capable of independent operation, but susceptible to injection locking with a signal applied to their outputs. In Fig. 1 two such oscillators are modeled as parallel equivalent circuits consisting of $L-C$ tanks and voltage-dependent negative conductances $-G_D(v)$. For most purposes a simple cubic function $-G_D(v) = -g_1 v + g_3 v^3$ suffices to model devices in systems with relatively small levels of injection power [5]. Circuit losses are modeled by conductances G_L, and in the absence of external loads ($I_1 = I_2 = 0$), each circuit will achieve a steady-state oscillation at a frequency $\omega_0 = 1/\sqrt{LC}$ and amplitude V_0 such that $-G_D(V_0) + G_L = 0$. In the discussion that follows, the oscillators are assumed to have identical characteristics.

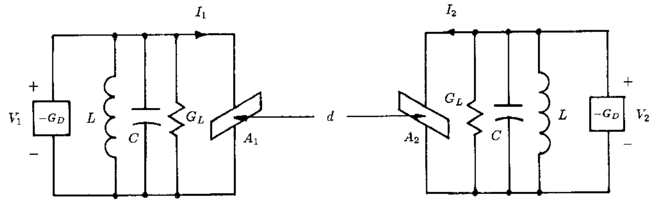


Fig. 1. Two identical oscillator circuits driving identical slot antennas separated by distance d.

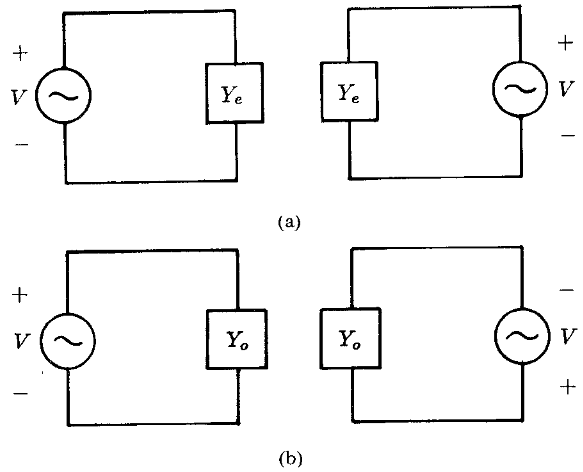


Fig 2 Oscillators of Fig. 1 treated by even-odd mode analysis: (a) even mode; (b) odd mode.

Consider the two antennas A_1, A_2 separated by a distance d and driven by the oscillators. Any pair of antennas can be represented by a mutual admittance matrix $[Y]$:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{1}$$

in which $Y_{21} = Y_{12}$ by reciprocity and $Y_{11} = Y_{22}$ in the present case by symmetry. Symmetry also informs us that the two normal modes of the system are (a) the even mode, in which $V_1 = V_2$, and (b) the odd mode, in which $V_1 = -V_2$. Substituting each expression into (1) gives us equivalent even-mode and odd-mode loads for the oscillators, which can now be treated independently as Fig. 2 shows:

$$\text{Even mode } Y_e = Y_{11} + Y_{21} \tag{2}$$

$$\text{Odd mode } Y_o = Y_{11} - Y_{21}. \tag{3}$$

The system analysis is now reduced to the simpler consideration of equivalent loads on independent oscillators.

A new and powerful technique known as the averaged potential method [5] has recently been applied to cylindrical microwave cavity power combiners [6]. For a system as simple as this one, a theorem for averaged potential theory is all that is needed to determine mode stability. This theorem states that in a system of coupled oscillators, "the oscillation changes in the direction of minimizing the time average of the dissipation in the entire system" [5]. This statement agrees with the intuitive reasoning behind mode-damping resistors placed so as to dissipate energy only when an undesirable mode is present, forcing a system to operate in a lower dissipation mode. In the case of the even and odd modes of Fig. 2, one merely calculates the steady-state dissipation from a knowledge of $-G_D(V)$, G_L, and $\mathrm{Re}[Y_e]$ or $\mathrm{Re}[Y_o]$. The mode resulting in the least dissipated power must be the stable mode.

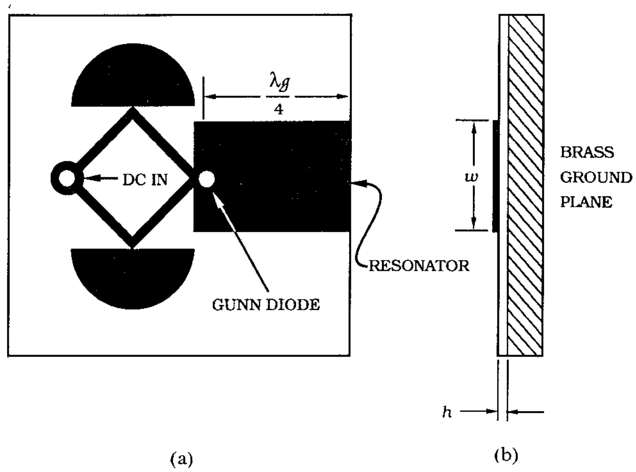


Fig 3 (a) Top view of layout of 10 GHz microstrip Gunn-diode oscillator. (b) Edge view.

III. Experiments

Since planar fabrication technology promises to exploit the interinjection-locking technique most fully, the experimental oscillators were designed around a microstrip quarter-wave resonator. Fig. 3(a) shows the dc bias network feeding the resonator at the low-impedance end, where the packaged diode was mounted so that its top cap could be soldered to the microstrip while the bottom portion was embedded in the 2.5 cm square brass ground plane. The high-impedance end of the resonator extended to the edge of the substrate, which coincided with the edge of the ground plane. Viewed edgewise in Fig. 3(b), the open microstrip end is seen to form a slot antenna of width $w = 0.5$ cm and height $h = 0.71$ mm, the thickness of the dielectric substrate (relative permittivity = 2.2). A simplified analysis was based upon the treatment of slot antennas by Derneryd [7]. He found that both the slot admittance ($Y_{11} = Y_{22}$ in our case) and the antenna directivity D could be expressed in terms of an integral in which the slot width w and the free-space wavelength λ appear:

$$I = \int_0^{\pi} \frac{\sin^2\left(\frac{\pi w}{\lambda}\cos\theta\right)\sin^3\theta}{\cos^2\theta}\, d\theta. \tag{4}$$

In terms of this integral, $Y_{11} = Y_{22} = (I/\pi)\sqrt{\epsilon_0/\mu_0}$ and directivity $D = (4\pi^2 w^2)/(I\lambda^2)$. By using a curve-fitting approximation to Derneryd's integral, we obtained these expressions for the admittance matrix elements:

$$Y_{11} = Y_{22} = \frac{\left(\frac{w}{\lambda}\right)^2}{(90\Omega)\left[1 + 1.78\left(\frac{w}{\lambda}\right)^2\right]^{1/2}} \tag{5}$$

$$Y_{12} = Y_{21} = -\frac{2w^2}{\eta\lambda d} e^{-j(kd+\theta)}. \tag{6}$$

The wavenumber $k = 2\pi/\lambda$, η = impedance of free space, and d is the interslot distance. The magnitude of the expressions for mutual admittance was obtained by assuming that antenna gain equals directivity (no losses). The phase factor was set equal to kd plus an unknown constant offset θ to be determined. Since these expressions are based upon far-field assumptions, they cannot be expected to apply for distances less than about a half-wavelength, but this restriction still allows most of the cases of practical interest to be treated.

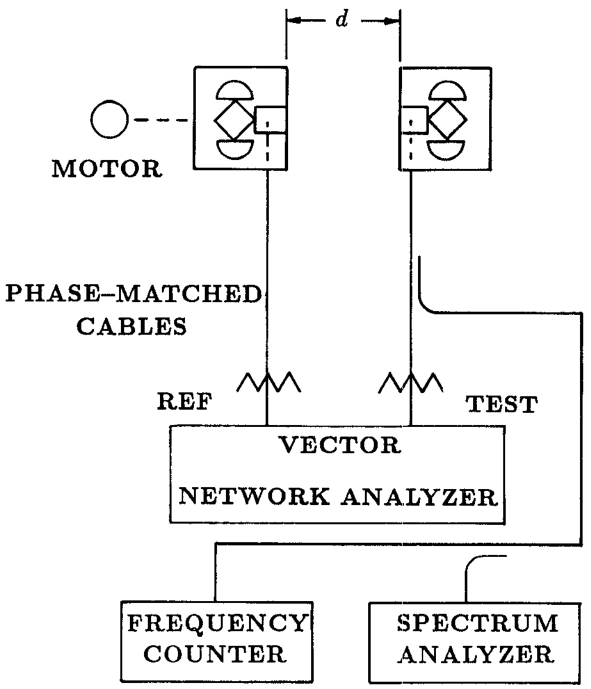


Fig. 4. Experimental setup of phase and frequency measurement versus separation of coupled oscillators.

An analysis of the microstrip oscillator circuit showed that over the relatively narrow frequency range of interest, it could be modeled by components $L = 0.215$ nH and $C = 1.178$ pF in Fig. 1 for a nominal free-running frequency of 10 GHz. Mode stability was determined by the fact that for all loads considered, the oscillator was undercoupled in the sense that an increase in the load admittance's real part led to an increase in output power. Therefore, the averaged potential theorem predicted that the mode having the smaller total load conductance was preferred. Since

$$\mathrm{Re}[Y_e] = Y_{11} - \frac{2w^2\cos(kd+\theta)}{\eta\lambda d} \tag{7}$$

$$\mathrm{Re}[Y_o] = Y_{11} + \frac{2w^2\cos(kd+\theta)}{\eta\lambda d} \tag{8}$$

we expected the circuit to alternate between even and odd modes as d increased, with a total period of one wavelength. The imaginary part of the coupling admittance was anticipated to cause a periodic deviation of the system's frequency about the nominal free-running frequency.

The experiment illustrated in Fig. 4 shows how the slot antennas faced each other as the separation was increased. Small coaxial probes beneath each resonator led to a vector network analyzer which displayed the relative phases between the voltages across each slot. The phase data were converted to an analog voltage, digitized, and stored by a computer which also controlled the translation stage that moved one oscillator with respect to the other.

The theoretical oscillator phase difference versus distance is shown in Fig. 5(a) in dashed lines, and is simply a square wave alternating between 0° (even mode) and 180° (odd mode). The experimental data in solid lines best fit the theory when the offset phase θ is chosen to be $-39°$, a relatively small correction which is probably due to effects of the reactive energy in the antennas' near fields. Agreement between theory and experiment is reasonably good overall. The experimental peaks in phase above 0° may arise from a dependence of oscillator reactance on output level, which was not modeled in our simple theory. The gaps between segments of data represent regions in which the oscillators were unlocked.

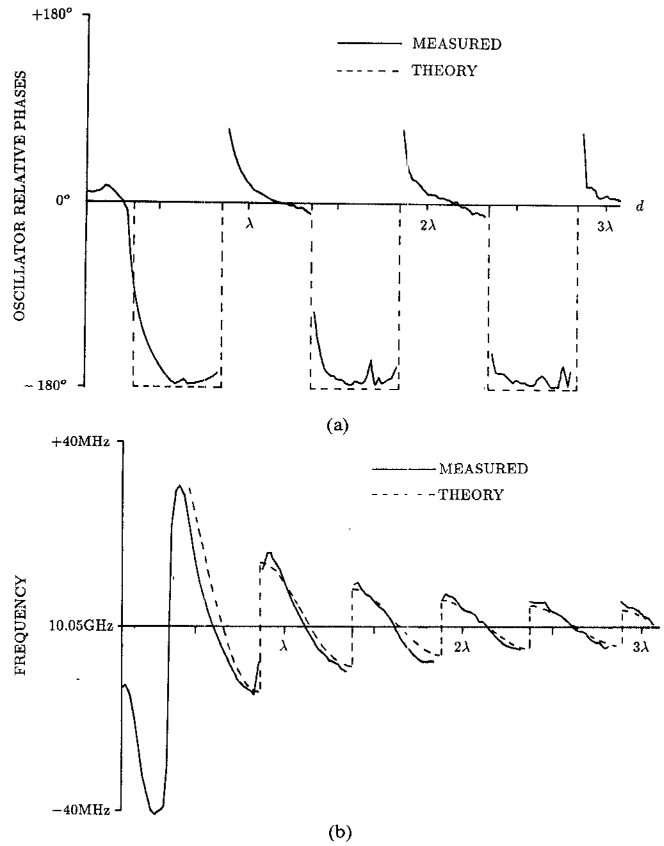


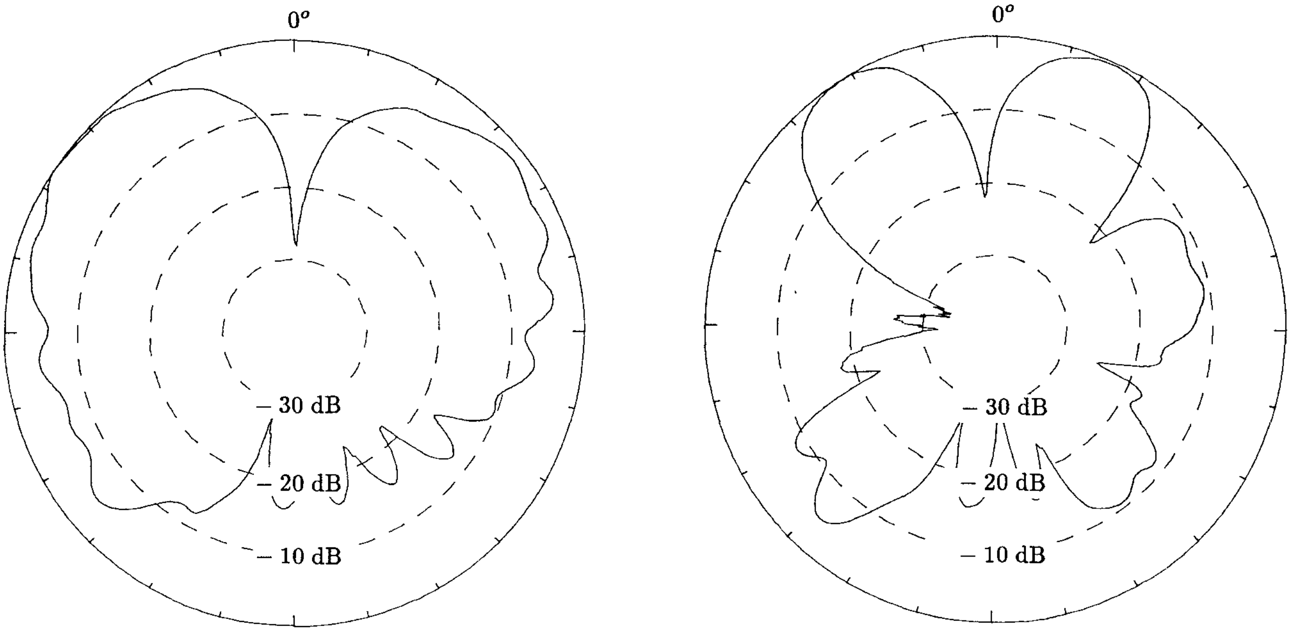
Fig. 5. Coupled oscillator measured and theoretical (a) phase and (b) frequency versus separation d.

The frequency data in Fig. 5(b) are presented in the form of frequency offset $f - f_0$ from a nominal center frequency $f_0 = 1/(2\pi\sqrt{LC})$. The theoretical frequency shift can be shown in the limit of small changes to be given by

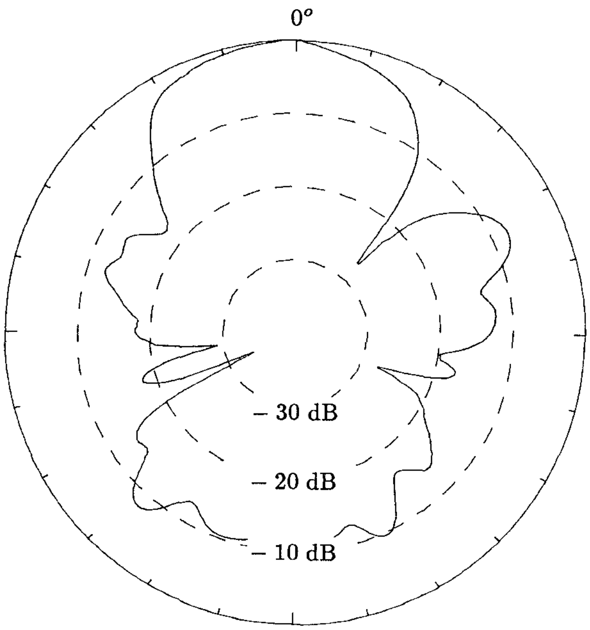
$$\frac{f-f_0}{f_0} \approx \pm\frac{w^2}{\eta\lambda}\frac{\sin(kd+\theta)}{d}\sqrt{\frac{L}{C}} \tag{9}$$

where the − sign applies to the even mode and the + sign to the odd mode. The $\sin(kd)/d$ behavior is followed quite closely by the experimental frequency data except where gaps appear during an unlocked condition, preventing measurement of a unique frequency. The theoretical model fails significantly only for distances less than $\lambda/2$, where we would expect the far-field analysis to break down in any case.

In a separate but related experiment, the E-plane radiation patterns of the two-element array were measured as a function of antenna separation. Fig. 6(a) shows a deep null at 0° as the receiving antenna views the even-mode oscillators head-on from the viewpoint of Fig. 4. Since the currents in the microstrip resonators are approximately equal in magnitude but opposite in direction, this null is predicted by symmetry considerations alone. At a spacing of approximately $\lambda/2$, the odd mode has taken over and the 0° null is replacing by a maximum, as shown in Fig. 6(b). Finally, as shown in Fig. 6(c), the even-mode null reappears at a spacing of approximately 1λ, but now occurs between two narrower lobes because of the increased distance between the antennas.



(a) (c)



(b)

Fig. 6 *E*-plane radiation patterns of coupled oscillators for separation d of (a) 0.184λ, (b) 0.478λ, and (c) 0.956λ.

IV. Conclusions

Finding the behavior of a system of oscillators coupled by radiation is in general a complex problem involving antenna theory, circuit theory, and nonlinear analysis. Nevertheless, by reducing the problem of radiation-coupled interinjection-locked oscillators to its simplest possible form, a two-element array, we have shown that fairly elementary theory can lead to satisfactory prediction of system behavior. It appears that the simple antenna mutual coupling analysis presented can be extended to larger arrays, and the average potential circuit model can be applied to larger systems.

It is encouraging to note that the array spacing of $\sim\lambda/2$ that gives a desirable broadside radiation maximum is of the same order as the typical distance between elements of a phased array. However, the fact that significant oscillator interaction was measured at distances exceeding 3λ means that future analyses of radiation-coupled arrays of this kind should take into account mutual coupling between all the elements within a few wavelengths of one another. With these precautions, radiation-coupled interinjection-locked systems promise to be a valuable addition to microwave and millimeter-wave power generation techniques.

References

[1] K. D. Stephan, "Inter-injection-locked oscillators for power combining and phased arrays," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 1017–1025, Oct 1986.

[2] K. D. Stephan and W. A. Morgan, Jr., "Analysis of inter-injection-locked oscillators for integrated phased arrays," *IEEE Trans. Antennas Propagat*, vol. AP-35, pp. 771–781, July 1987.

[3] R. J. Dinger, D. J. White, and D. R. Bowling, "10 GHz space power combiner with parasitic injection locking," *Electron. Lett.*, vol 23, pp. 397–398, Apr. 9, 1987.

[4] K. Kurokawa, "The single-cavity multiple-device oscillator," *IEEE Trans Microwave Theory Tech*, vol MTT-19, pp. 793–801, Oct 1971.

[5] M. Kuramitsu and F. Takasi, "Analytical method for multimode oscillators using the averaged potential," *Elec. Commun. Japan*, vol. 66-A, pp 10–19, 1983.

[6] K. Fukui and S. Nogi, "Mode analytical study of cylindrical cavity power combiners," *IEEE Trans. Microwave Theory Tech*, vol MTT-34, pp. 943–951, Sept. 1986.

[7] A G. Derneryd, "Linearly polarized microstrip antennas," *IEEE Trans. Antennas Propagat.*, vol. AP-24, pp. 846–851, Nov. 1976.